PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

The Free Librations of a Dissipative Moon [and Discussion]

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Phil. Trans. R. Soc. Lond. A 1981 **303**, 327-338
doi: 10.1098/rsta.1981.0206

Phil. Trans. R. Soc. Lond. A **303**, 327–338 (1981)
Printed in Great Britain

The free libations of a dissipative Moon

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Dissipation in the Moon produces a small offset, *ca.* 0.23″, of the Moon's rotation axis from the plane defined by the ecliptic and lunar orbit normals. Both solid body tidal friction and viscous fluid friction at a core–mantle interface are plausible mechanisms. In this paper, I discuss the merits of each and find that solid friction requires a low lunar tidal Q, *ca.* 28, while turbulent fluid friction requires a core of radius *ca.* 330 km to cause the signature observed by lunar laser ranging. Large (*ca.* 0.4–8.0″) free librations of the lunar figure have also been detected by laser ranging. Both a very recent impact on the Moon and fluid turbulence in the lunar core are plausible mechanisms for generating these free librations.

Introduction

The Apollo programme's greatest contribution in the field of dynamical astronomy resulted from the placement of three cube corner retroreflectors by the astronauts of Apollo 11, 14 and 15. Laser ranges to these reflectors and the Russian reflector on Lunakhod 2 have been regularly obtained at McDonald Observatory in Texas over the last decade. The modelling of these precise measurements has led to improved theories of Earth rotation, lunar orbital motion and lunar figure rotations or physical librations accurate to a few tens of centimetres. The essential ingredients of this experiment's success are its highly accurate, *ca.* 10 cm range measurements coupled with its long time baseline. In addition, laser ranging has provided significant tests of gravitational theories, the equivalence of inertial and gravitational mass (Nordtvedt effect) and an estimate of the tidal acceleration of the Moon in its orbit (see Mulholland (1980) for references).

One of the unexpected results of this experiment has been the detection of a libration signature indicative of dissipation within the Moon (Yoder *et al.* 1979; Yoder 1979; Capallo 1980; Ferrari *et al.* 1980). The two possible mechanisms that could produce this signature are: (1) anelastic tidal flexing of the Moon by the Earth and (2) viscous fluid friction caused by the relative motion of a small fluid core with respect to its envelope. The principal focus of this discussion is to investigate the relative merits of these two mechanisms. It is found in fact that if the Moon has a 330 km radius core then turbulent fluid friction and not solid friction is the primary mechanism affecting the forced lunar librations.

Another measurement that appears to be incompatible with significant dissipation in the Moon is the detection of large free librations of the lunar figure (Calame 1977). I discuss the problem of excitation versus damping. The possible mechanisms for generating the free motions are moonquakes, impacts (possibly recent) of stray asteroidal or cometary bodies and precession driven fluid turbulence. It is found that only a very recent impact (Calame & Mulholland 1978) or turbulent friction is compatible with both 'large' librations and 'large' dissipation.

Laser ranging and the lunar spin axis offset

I shall begin by briefly describing the laser ranging experiment. A short laser pulse is fired from McDonald Observatory towards one of four cube corner retroreflectors placed on the Moon. After about 2.5 s if the aiming and atmospheric seeing are good, then maybe one of the 10^{18} photons in the laser pulse is electronically detected back at McDonald. Ten or so successful firings out of several hundred attempted are converted into a normal point or range. After corrections for atmospheric refraction, relativistic clock corrections and adjustments for the motion of Moon and Earth during the 2.5 s, the two-way timing measurement can be converted to a one-way range measurement with typical accuracy of about 10 cm. The range ρ from McDonald to a reflector,

$$\rho = |\boldsymbol{r} - \boldsymbol{R}_{\oplus} + \boldsymbol{R}_{☾}| \approx r + \boldsymbol{R}_{☾}\cdot\hat{\boldsymbol{r}} - \boldsymbol{R}_{\oplus}\cdot\hat{\boldsymbol{r}},$$

can be separated into a part equal to the distance of separation r of the centre of mass of Earth and Moon and parts that depend on the projection of the orbit vector $\boldsymbol{r}$ onto the Earth vector $\boldsymbol{R}_{\oplus}$ describing the position of the observatory relative to the Earth's centre of mass and onto the lunar vector $\boldsymbol{R}_{☾}$ describing the position of a reflector.

The range contains information about the spacial orientation of the Earth and lunar figures relative to the lunar orbit vector $\boldsymbol{r}$ from the two projections, $\boldsymbol{R}_{\oplus}\cdot\hat{\boldsymbol{r}}$ and $\boldsymbol{R}_{☾}\cdot\hat{\boldsymbol{r}}$. The apparent latitude of the Moon as seen at the observatory is determined from that part of the range which depends on the observatory R_z coordinate:

$$\boldsymbol{R}_{\oplus z}\cdot\hat{\boldsymbol{r}} \approx R_{\oplus z}[\sin 23.4^\circ \sin L + \sin 5.2^\circ \sin (L-\Omega)],$$

where 23.4° is the Earth's obliquity, 5.2° is the orbit inclination, L is the lunar orbital longitude and Ω is the node of orbit. The retrograde circulation of the node Ω ($2\pi/\dot{\Omega} = -18.6$ a†) separates the obliquity- and inclination-dependent contributions to the range.

The apparent latitude of the Earth as seen from the Moon comes from that part of the range which depends on $R_{☾z}$ of the reflector:

$$\boldsymbol{R}_{☾z}\cdot\hat{\boldsymbol{r}} \approx R_{☾z}[\sin 5.2^\circ \sin (L-\Omega) + \sin 1.5^\circ \sin (L-\Omega-\Delta\phi)].$$

The part proportional to $\Delta\phi$ describes the possible offset of the lunar spin axis from the Cassini state. In the absence of internal dissipation caused by either tides or fluid core–mantle friction, the lunar spin axis and orbit normal precess at the same rate about the mean ecliptic normal but 180° out of phase. Dissipation in the Moon causes a small negative offset, $\sin 1.5^\circ \Delta\phi \approx -0.23''$, as shown in figure 1. This offset is observed as a small, metre-sized, monthly variation in the range proportional to $\cos (L-\Omega)$ and appears to be a unique signature of dissipation, whether it results from solid body friction or viscous core–mantle friction. The counterbalancing torques exerted by the Earth on the second and third degree harmonic gravity fields of the Moon do cause a shift in the coordinates of the reflectors (which are tied to the second harmonic field) but do not cause a shift of the spin axis in space. Additional confidence in the resulting measurement of the offset is obtained from the fact that the libration signatures for the four reflectors differ because of their dependence on the $R_{☾z}$ coordinate of each reflector. Of course, it is just this differential signature that helps separate the orbital variations from the optical and physical librations of the lunar

† Symbols a and d stand for years and days respectively.

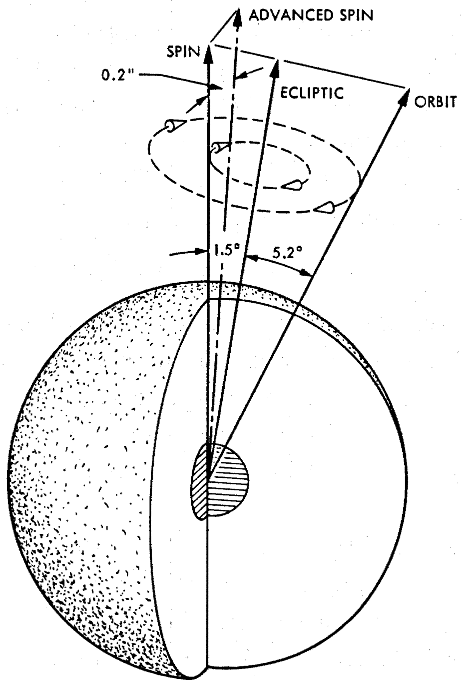

FIGURE 1. The lunar rotation axis precesses about the ecliptic normal and *ca.* 180° out of phase with the precessing orbit normal. Dissipation in the Moon causes a small 0.2″ advance in the lunar rotation axis.

figure. For solid body friction, the latitude offset is related to the (k/Q) factor for the Moon by the relation

$$\sin 1.5^\circ \, \Delta\phi = -223'' \, k/Q.$$

The factor Q^{-1} is the fraction of tidal strain energy dissipated per flexing cycle, and k is the second harmonic Love number.

Solid friction also causes an offset in the longitude of the lunar figure *ca.* 390″ (k/Q), but cannot be separated from the (inferred) offset caused by the third harmonic field or even a simple rotation of the longitudinal coordinates of the reflectors based on an analysis of only laser ranging data. Appraisal of this effect would require extremely accurate lunar orbiter data to determine both the magnitude of the gravity field and the spatial orientation of the principal axis system with respect to the lunar orbit.

There are several shortcomings in the laser ranging data set. First of all, no ranges are obtained in a 4–6 d band about new moon. This limitation is imposed by the present inability to accurately aim the laser pulse at a reflector without visible tracking features. Thus signatures with frequency ν can alias into signatures with frequencies $\nu \pm$ the lunar synodic frequency of $2\pi/(29.5\,\mathrm{d})$. Secondly, we presently rely on the values obtained by B.I.H. for the polar motion of the Earth's spin axis with respect to the Earth-fixed coordinate frame. The estimated error of this input is of order 40 cm and since there is only one reliable ranging station these errors can affect the solution for the orbit and lunar librations. Finally, about 80 % of the ranges have been to just one reflector (at Hadley), decreasing our ability to separate orbital and libration signatures. Still, I believe that the observed offset in latitude is real and not due to either data or model deficiencies. This view is based on the uniqueness of the signature and the significant reduction in the range residuals when this effect is included in the model.

Instead of simply solving for a spin axis offset from the range data, we introduce a time lag T into the otherwise elastic deformation by the Earth of the lunar moment of inertial tensor. The solution parameter is kT. Theoretical estimates for the lunar k are *ca.* 0.029 (Cheng & Toksoz 1978) while the latest solution value is *ca.* 0.022 ± 0.011 (Ferrari *et al.* 1980). The uncertainty in the solution for kT is actually smaller than that for k alone. The time lag T is related to Q and the 27.3 d orbit period by the relation $Q = (27.3\,\text{d}/2\pi T)$. The latest solution for k/Q based on a least squares solution of about 3000 ranges spanning 9 a is

$$(k/Q) = (1.04 \pm 0.12) \times 10^{-3}$$

(Dickey *et al.* 1980). The corresponding time lag is *ca.* 4 h. Capallo *et al.* (1981) find $k/Q = (1.08 \pm 0.05) \times 10^{-3}$ from an independent analysis of ranging data. Their error estimate is about three times the formal uncertainty whereas the error quoted by Dickey *et al.* is five times the formal value. The different magnification factors chosen by each group reflect independent judgements of the possible influence of model deficiencies and systematic effects which may degrade the least squares solution.

A significantly larger value, $k/Q = (1.66 \pm 0.23) \times 10^{-3}$, was obtained from a combined solution of 7 a of lunar laser ranging data and Doppler tracking data from Lunar Orbiter 4 (Ferrari *et al.* 1980). The Doppler data are more sensitive than laser ranging to most components of the lunar gravity field with degree $\geqslant 3$. Apparently, the libration signatures caused by some of the third harmonic components of the lunar gravity field are sufficiently correlated with the offset signature over the 7 a of ranging to raise the nominal value of k/Q obtained from analysis of only ranging data. The laser ranging solution for k/Q is now considered more reliable.

Possibly the most disturbing aspect of this result is that the solid body lunar Q is small compared to our best guess of a few hundred for a tidal Q and lunar seismic $Q \approx 10^3$. For $k = 0.029$, $Q = 28 \pm 3$. The Earth's tidal Q is smaller (*ca.* 13), but the principal source of tidal dissipation is in the Earth's oceans and not in the solid Earth. A reasonable lower bound on the Earth's solid tidal Q of *ca.* 60 at a 435 d flexing period can be obtained from the *ca.* 25 a damping time for the Chandler wobble although here again dissipation in oceans may be the cause (Wunsch 1974). Estimates of the Martian Q obtained from the secular acceleration of Phobos in its orbit fall in the range: $50 \leqslant Q_{☿} \leqslant 150$ (Smith & Born 1976). Thus the expected value for the lunar Q is about 2–10 times the Q inferred from the observed offset if it is entirely attributed to solid friction.

One can, of course, ask if there is any evidence, such as its interior structure, that would suggest the source of large solid dissipation in the Moon. The basic structure of the Moon derived from seismic data includes a 60 to 100 km crust of mean density *ca.* 2.8 g/cm^3, a nearly uniform mantle structure, a seismically attenuating zone and possibly a liquid core. In the (Goins *et al.* 1979) model the upper and lower mantle are separated by a 100 km middle mantle or transition zone. Nakamura *et al.* (1974) derive a similar structure, although the transitions within the mantle tend to occur at shallower depths. The biggest difference is that they find a thickness of 300 km for the transition zone. The seismic velocities appear to be remarkably constant within the mantle, decreasing by less than 0.2 km/s in going from the upper to the lower mantle, and imply a nearly uniform mantle density of *ca.* 3.4 g/cm^3. The shear wave Q is *ca.* 5000 in the crust and *ca.* 3000 in upper mantle and drops to *ca.* 1500 in the lower mantle (Goins *et al.* 1979). The high seismic Q values are attributed to the absence of water or other volatiles and partial melt (Tittman *et al.* 1976). Deep focus moonquakes occur in the lower mantle at depths of 700 to 1100 km and are apparently controlled by lunar tides (Toksoz *et al.* 1977).

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Binder (1980*a*) argues that the apparent frontside bias in epicentres can be explained in terms of a low Q of *ca.* 200 below 1000 km depth and the assumption that number of epicentres per unit area is proportional to the tidal potential. If his argument is correct then these moonquakes release normal rather than shear tidal stress. The deep focus moonquakes release *ca.* 10^8 J/a compared with *ca.* 6×10^{16} J/a from solid friction, if one assumes that $(k/Q)_{☾}\approx 1\times10^{-3}$. Obviously this seismic mechanism is an insignificant sink of tidal energy. Below a depth of 900 to 1100 km lies an attenuating zone with low seismic Q of *ca.* 100. The low Q is believed to be due to the presence of partial melt. If one attempts to bury most of the solid friction within a 700 km 'soft' region then the tidal Q within that region would be *ca.* 2! Thus there is no evidence from internal structure that accounts for a bulk Q as low as 28.

According to Nakamura *et al.* (1974), the Moon may have a small (170–360 km radius) core. Their evidence is the delay in arrival time from a single, backside, impact event. Goins *et al.* (1979) tend to dismiss this datum and claim that the arrival time uncertainty is of the same order as the observed time delay. The observed mean moment of inertia ($I = (0.3905\pm0.0023)MR^2$; Ferrari *et al.* 1980) is consistent with an iron ($\rho_c = 7.0\,\mathrm{g/cm^3}$) core of radius 400 km (Levin 1979), although the uncertainty in the moment estimate is such that a high density core may be unnecessary. However, Binder (1980*b*) finds that a 200–400 km radius iron or iron-rich core is indicated from an analysis of the uncertainties of the moment of inertia, the radial density, thermal structure and composition. No highly conducting iron core of radius > 400 km has been detected from magnetometer data (Goldstein 1979). It is unknown whether there is enough internal heat to maintain a liquid iron core. The absence of an internal magnetic field may be an argument against its being liquid. However, Stevenson & Yoder (1981) find that only a small amount of sulphur is required in a primordial core melt to maintain a liquid Fe–S entectic layer to the present day. As the iron freezes onto a solid core, the sulphur is concentrated into an outer liquid core, thereby depressing the melting temperature of the mixture below the minimum expected temperature near the moon's centre. Although the existence of a primordial core is uncertain, we can at least ask what constraints are imposed on a liquid lunar core if we attribute the 0.23″ offset to viscous friction at the core–mantle interface.

The offset and core–mantle friction

The mechanisms that tend to couple the motion of a fluid core with its mantle envelope include the conservative Poincaré pressure torque which is proportional to the ellipticities of the envelope and the dissipative viscous stress within a thin boundary layer inside the envelope. The pressure torque is the primary mechanism affecting the Earth's fluid core and causes the Earth's core to closely follow the slow nutations of the mantle. It is unlikely that the pressure torque is an important coupling mechanism for a lunar core since it would require a large non-hydrostatic ellipticity of *ca.* 0.01 to force the core to follow the 18.6 a nutation of the lunar mantle.

The simplest conceivable model for core–mantle friction is that the torque T_v acting on the mantle depends on the differential angular velocities of the bulk fluid core ω_c and rigid mantle envelope ω_m,

$$T_v = K(\omega_c - \omega_m). \tag{2}$$

If the coupling involves a thin laminar boundary layer, the parameter K equals $2.6\,\omega_c I_c(\nu/R_c^2\omega_c)^{\frac{1}{2}}$ for the 'spin-over' mode (see, for example, Greenspan 1968). The offset $\sin\epsilon\,\Delta\phi$ (ϵ is the lunar obliquity) is related to the ratio of the core to mantle mean moments of inertia, I_c/I_m, and dimensionless coupling parameter $\xi = K/I_c\dot{\Omega} = 2.6(\nu/R_c^2\omega_c)^{\frac{1}{2}}(\omega_c/\dot{\Omega})$ by

$$\sin\epsilon\,\Delta\phi = 1.3\,\frac{I_c}{I_m}\,\frac{\xi\sin\epsilon}{1+\xi^2}. \tag{3}$$

If the $-0.23''$ latitude offset is attributed to viscous core–mantle friction, a constraint on core size, density and viscosity is obtained through (3). The minimum core moment that can produce this offset is $6.5\times10^{-5}\,I_m$. For a 7.0 g/cm³ core density the minimum radius is *ca.* 220 km and the required viscosity is *ca.* 3300 St (1 St = 1 cm²/s). The Earth's fluid core viscosity is estimated to lie in the range 0.004 to 0.02 St (Gans 1972) although the observational upper bound is *ca.* 10^5 St (Toomre 1974). As the core radius is varied, the required viscosity to satisfy (3) can rapidly decrease or increase depending on whether the core–mantle coupling is weak (i.e. $|\xi| < 1$) or strong (i.e. $|\xi| > 1$), respectively (see figure 2). Weak coupling implies that the core spin axis tends to remain closer to the ecliptic normal than the precessing mantle normal while for strong coupling the core spin axis is more nearly coincident with the precessing mantle.

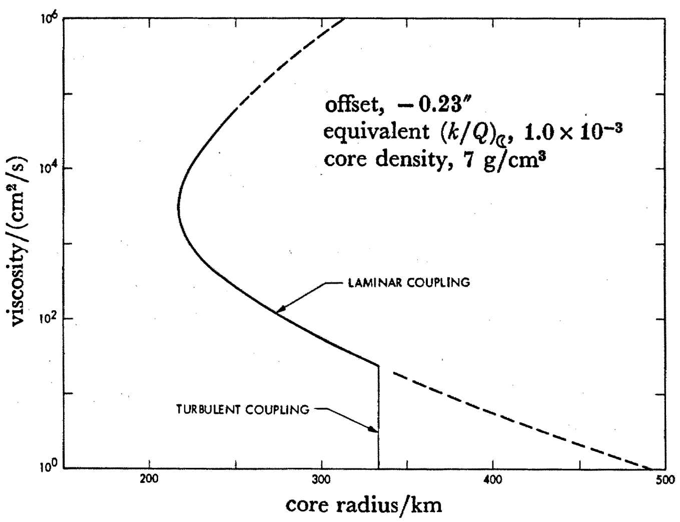

FIGURE 2. Graph of kinematic viscosity against lunar core radius which causes *ca.* 0.23″ offset of the mantle spin axis. The solid part of the curve represents the allowed range of viscosities.

The libration signature in the limit of strong coupling can be significantly different from a simple offset. The absence of a strong coupling signature bounds $\nu \lesssim 2\times10^4$ St and $R_c \lesssim 250$ km.

There exists another bound in the weak coupling limit based on the onset of fluid turbulence. Once turbulence sets in, it is expected that the rate of dissipation will tend to be independent of the molecular viscosity and depend on an 'eddy' viscosity. The problem of turbulent core–mantle coupling has yet to be resolved either theoretically or experimentally. For the Moon, surface roughness at the core–mantle boundary may play a dominant role.

A reasonable estimate of the turbulent couple can be obtained from the skin friction approximation where the local surface stress $\boldsymbol{\tau}$ is proportional to the square of the shear velocity $\boldsymbol{u}$ at the interface. (The explicit formula is $\boldsymbol{\tau} = 0.002\,\rho|\boldsymbol{u}|\boldsymbol{u}$ (Bowden 1953).) If $\boldsymbol{u}$ is set equal to $\boldsymbol{r}\times(\boldsymbol{\omega}_m-\boldsymbol{\omega}_c)$ and the local torque $\boldsymbol{r}\times\boldsymbol{\tau}$ is integrated over the sphere, the equivalent K_t in (2) equals $0.0088\,I_c\,|\omega_m-\omega_c|$. For weak coupling $|\omega_m-\omega_c| \approx \omega_c \sin\epsilon$ and the turbulent coupling parameter,

$$\xi_t \approx 0.0088\,(\omega/\dot{\Omega})\sin\epsilon = -0.059, \tag{4}$$

is independent of core radius. Given $\rho_c = 7\,g/cm^3$, from (3) $R_c = 330$ km. The equivalent viscosity for laminar flow is *ca.* 20 St. Thus the couple is laminar only if $\nu \gtrsim 10^2$ St. This is well outside the expected range of core viscosities. Therefore, if the Moon has a fluid core and the core is principally responsible for the latitude offset, then the coupling is turbulent and the core radius is expected to be close to 330 km.

Presently, the numerically integrated laser ranging model developed at J.P.L. only includes dissipation due to solid friction. Implementation of a model involving core friction is not attempted since the expected differential signature is 10% or less than that of the latitude offset of 0.2″, at least for a 'weak' couple between core and mantle.

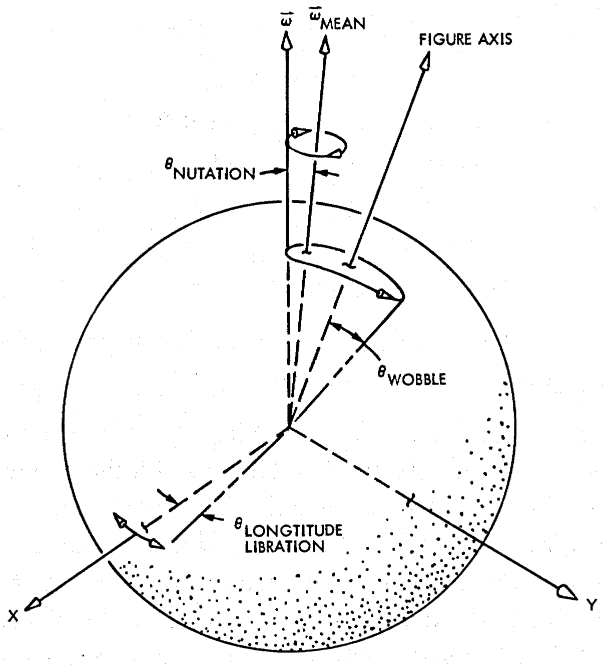

FIGURE 3. The three possible free librations of the lunar figure are a longitude libration, wobble and nutation.

Free librations

In addition to measuring the latitude offset, laser ranging can also be used to determine the amplitude and phase of the three free librations of the lunar figure (figure 3). The three possible free motions include a 2.89 a libration in longitude which is symmetric about the spin axis and results from the torque exerted by the Earth on the lunar B–A moment difference. The remaining two librations, a wobble and a nutation, affect the apparent latitude of a reflector. The Chandler-like wobble results from non-principal axis rotation, and the spin axis traverses a prograde elliptical path about the principal figure axis with a period of 74.3 a. The remaining motion, a free nutation or retrograde precession of the spin axis in space, has a period of 80.1 a and is detected by laser ranging as a near-monthly variation in the range. Unfortunately, the amplitude and phase of free motions are not explicit solution parameters in a numerically integrated model.

The method employed by Calame (1977) and Capallo (1980) is to obtain an ephemeris of the figure motions using an analytic solution generated by Eckhardt (1981) which is based on the same set of constants and initial conditions as the numerically generated ephemeris. If the differential ephemeris has signatures that can be identified with the free librations, one can be reasonably certain that the residual motion represents the free librations. The major uncertainty with this approach is that there may be forcing terms with periods very near the free motions that are inadequately modelled in the analytic theory. Probably the most serious deficiency in this respect is the reliance in Eckhardt's (1981) libration model on Brown's (I.L.E. 1954) theory for the planetary and additive terms in the theory of the lunar orbit which act as drivers on the lunar figure librations.

The results obtained by Calame (1977) for the free librations (LLB 5 model) are substantial compared with the *ca.* 0.01″ sensitivity of the ranging data:

longitude libration

$$\theta_{\mathrm{L}} = 1.8'' \sin(\phi_{\mathrm{L}}(t) + 39^\circ); \tag{5}$$

wobble (body-fixed)

$$\theta_{\mathrm{W}x} = 3.0'' \cos(\phi_{\mathrm{W}}(t) + 105^\circ)$$
$$\theta_{\mathrm{W}y} = 7.8'' \sin(\phi_{\mathrm{W}}(t) + 105^\circ); \tag{6}$$

nutation (space-fixed)

$$\theta_{\mathrm{N}x} = 0.4'' \cos(\phi_{\mathrm{N}}(t) + 4^\circ)$$
$$\theta_{\mathrm{N}y} = 0.4'' \sin(\phi_{\mathrm{N}}(t) + 4^\circ). \tag{7}$$

The argument $\phi(t)$ equals $\sigma(t-t_0)$ where σ is the appropriate libration frequency

$$(\sigma_{\mathrm{L}} = 2\pi/(2.89\,\mathrm{a}), \quad \sigma_{\mathrm{N}} = 2\pi/(80.1\,\mathrm{a}) \quad \text{and} \quad \sigma_{\mathrm{W}} = -2\pi/(74.3\,\mathrm{a})).$$

The epoch t_0 = 2440400.5 d and the phase is in east longitude, measured from the sub-Earth point. Calame tried various models that included additional solution parameters and found that the phase and amplitude of the wobble and longitude libration were stable to within 5%. However, the free nutation was considerably less stable and could have nearly zero amplitude.

R. J. Capallo (private communication) has obtained similar results based on a comparison of his numerically generated solution obtained from an analysis of 9 a of ranging with Eckhardt's (1981) 500 series semi-analytic solution. Capallo also finds that the post-fit residuals are 0.2–0.5″ in amplitude indicating that Eckhardt's theory inadequately models the forced motion of the lunar figure at this level.

The energy in the wobble, nutation and longitude libration are 2×10^8, 10^{10} and 10^{10} J, respectively. These energy estimates are based on the amplitudes in (5)–(7), are of the same order as the 10^9 J of seismic energy released per year by lunar moonquakes and are also much smaller than the maximum possible energy dissipated by either solid friction (6×10^{16} J/a) or core–mantle friction (5×10^{15} J/a). On this basis, the observed amplitudes of the free motion appear plausible. It is only when we make a detailed comparison of excitation mechanisms with the damping rates by either solid or viscous friction that we begin to appreciate their significance.

Let us first consider the time scales imposed by the two damping mechanisms. The 1.5° forced nutation and free nutation of the lunar spin axis flex the Moon at near-monthly periods and can be expected to have similar values of Q. However, the longitude libration and wobble periodically flex the Moon in 2.9 and 74 a respectively and their values of Q may be different from that at 1 month. Both solid and viscous fluid friction damp the amplitude θ according to an exponential law: $\theta(t) = \theta(0)\exp(-t/\tau)$. If we assume that solid friction causes the offset and that Q is independent of frequency, we find that the damping time τ for each of the free motions is

$$\tau_L = 10^4, \quad \tau_W = 4\times10^5 \quad \text{and} \quad \tau_N = 4\times10^5\,\text{a} \tag{8}$$

(Peale 1976; Yoder & Ward 1979). The damping times are inversely proportional to k/Q and have been evaluated with $k/Q = 10^{-3}$. It is unlikely that the actual solid friction τ values are larger than *ca.* 20 times the estimates in (8), even if solid friction is not the primary cause of the latitude offset.

The damping times due to fluid friction are

$$\tau_L = 6\times10^5, \quad \tau_W = 2\times10^8 \quad \text{and} \quad \tau_N = 3\times10^5\,\text{a}, \tag{9}$$

if we assume that the latitude offset of $-0.23''$ primarily results from the core–mantle interaction. Clearly, wobble damping is primary controlled by solid friction rather than viscous friction, and τ_W should be less than *ca.* 10^7 a if we take $Q \approx 500$ as an upper bound. The upper bounds on τ_L and τ_N are *ca.* 2×10^5 and 3×10^5 a respectively, one-thirtieth of the upper bound on τ_W. It is found that, based on Peale's (1976) analysis, mean times between impacts that result in the libration amplitudes given in (5)–(7) are 3×10^5 to 3×10^6 a for the longitude libration, 10^6 to 10^7 a for the nutation and 10^7 to 10^8 a for the wobble. Thus it seems unlikely that the observed free motions are the result of impacts, unless we are observing the Moon at a special point in time just after a large impact.

Excitation mechanisms

There are three excitation mechanisms that could plausibly excite the free librations of the lunar figure; moonquakes, impacts and turbulent core–mantle friction. On the Earth, earthquakes are a plausible, if not proven, source mechanism for generating the Chandler wobble. Earthquakes excite the wobble by inducing random variations in the moment of inertia tensor, thereby causing a near-instantaneous shift of the principal polar axis with respect to the spin axis. The energy associated with seismic activity is *ca.* 10^{10} smaller on the Moon than on Earth, rendering this an unlikely mechanism for excitation.

I have already remarked that impacts are also an improbable excitation mechanism given the imperfect knowledge of impact rates and the dynamics of crater formation. Although improbable, excitation by a recent impact is testable in that the relative age of the major lunar craters can be established by analysing the photogeological record. Hartung (1976) has hypothesized that the lunar crater Giordano Bruno (103° E, 36° N) was catastrophically formed on the evening of 18 June 1178 (Julian date = 2 151 491.4 d). As evidence he cites the chronicle of Gervase of Canterbury which reports unusual lunar phenomena observed by several eye witnesses which can be interpreted in terms of an impact just behind the limb of the moon. Crater Bruno fits the description in that the crater has an extensive ray system (indicating relative youth) and is in the right location. Calame & Mulholland (1978) argue that this recent event may have excited the observed librations, although they admit that the inferred angular momentum impulse and change in the moment is not large enough to account for the observed wobble.

If a recent impact is responsible for the observed free librations reported by Calame (1977), then that impact is constrained to predict both the observed amplitudes and phases. The amplitude of the libration can be estimated from the known crater size (*ca.* 20 km), a reasonable guess at impact velocity (*ca.* 20 km/*s*), impact energy versus crater diameter scaling laws and a model of the crater, its rim and ejecta blanket, and finally an estimate of the amount of material added or ejected into space (Peale 1975). Gault *et al.* (1966) estimate that *ca.* 1 %. of the mass ejected from the crater is above escape velocity and is lost into space. This mass loss is much greater than the mass added by the impacting projectile. The 1 % mass loss tends to dominate the excitation of the wobble for crater diameters $\lesssim$ 30 km. If there were no free librations before impact, then both the displacement of material away from the crater and the mass loss would tend to displace the principal polar axis toward the crater, along its longitude. The vertical displacement of material displaces the principal axis in the opposite direction and dominates over the lateral displacement for crater diameters $\lesssim$ 5 km (Peale 1975).

The component of the angular momentum impact impulse perpendicular to the spin axis tends to move the spin axis away from the polar figure axis and along a longitude 180° away from the crater longitude. Thus the initial phase of both the wobble and nutation are well determined. The libration phase is determined to within ± 180° if the excitation resulting from the parallel component of the angular moment impulse dominates over that caused by the shift in the principal axes. Unfortunately, the uncertainty in the short longitude libration period causes the libration phase after 791 a to be uncertain to more than one cycle.

Calame & Mulholland (1978) obtain the following estimates of the amplitudes for the Bruno event, based on Peale's (1975) model:

$$\left.\begin{aligned} 0.2'' \leqslant \theta_{\mathrm{L}} \leqslant 4.6''; \\ 0.014'' \leqslant \theta_{\mathrm{W}} \leqslant 0.2''; \\ 0.006'' \leqslant \theta_{\mathrm{W}} \leqslant 0.14''. \end{aligned}\right\} \tag{10}$$

The extremes in amplitudes are obtained from two different energy versus diameter scaling laws and hopefully represent the model uncertainty.

The expected wobble excitation is at least an order of magnitude smaller than the observed value (6). A less model-dependent test is the wobble phase. The initial wobble phase predicted at impact is 103° + 180° = 283°. The wobble has executed 10.65 oscillations between the time of impact and the epoch defining the phase of the observed wobble. The predicted phase at epoch is *ca.* 157° as compared with the observed phase of 105°. This argument appears to eliminate the Bruno event as the source of the wobble.

The third candidate for excitation is precession-driven turbulence. This mechanism is the least well understood. I shall sketch some crude arguments to indicate the necessary eddy size, the expected random fluctuation in the position of lunar spin axis and the relative amplitudes of the free motions if this mechanism generates the observed free librations. For the sake of argument, let us assume that the fluid core has M eddies with scale size $l \approx (4\pi R_{\mathrm{c}}^2/M)^{\frac{1}{2}}$ in a turbulent boundary layer of thickness l which are generated by the 18.6 a forced nutation of the lunar spin axis. The eddy scale length may arise either from the scale $\sin\epsilon\, R_{\mathrm{c}} \approx$ 5–10 km associated with the monthly shear at the core–mantle interface or from the (unknown) scale of surface roughness. Let us assume that the number of eddies controls the magnitude of the turbulent torque and that random fluctuation of the number of eddies is *ca.* $M^{\frac{1}{2}}$. This suggests that the random fluctuation in the core–mantle torque is *ca.* $T_{\mathrm{v}}/M^{\frac{1}{2}}$. The time scale for the fluctuation in T_{v} may be as short as 1 month or as long as the turnover time τ_{eddy} of the eddies *ca.* $2\pi l/u_{\mathrm{eddy}}$, where u_{eddy} is the typical eddy velocity of order $\xi_{\mathrm{t}} u_{\mathrm{shear}} \approx$ 0.2 cm/s.

The excitation of the free nutation will depend on the spectral power of the fluctuation near its period of 80.1 a. For a stationary random process the mean free nutation amplitude should be approximately equal to $(\tau_N \sigma_N)^{\frac{1}{2}} = (2\pi\tau_N/80\,a)^{\frac{1}{2}}$ times the fluctuation in the offset $0.23''/M^{\frac{1}{2}}$ or $\theta_N \approx 0.23''\,(2\pi\tau_N/80\,M\,a)^{\frac{1}{2}}$. For $\theta_N \approx 0.4''$ and $\tau_N \approx 3\times10^5$, $M \approx 6000$, the eddy scale size $l \approx 10$ km and $\tau_{eddy} \approx 1$–2 a.

The time scale τ_{mag} for diffusion of a magnetic field generated by the fluid turbulence is of order $4\pi\mu\sigma l^2$ or about 1 a if $\sigma \approx 3\times10^{-6}$ e.m.u. The fact that $\tau_{eddy} \approx \tau_{mag}$ may explain why precession driven turbulence is apparently an ineffective mechanism for generating a lunar magnetic field by dynamo action.

The wobble is excited by the random fluctuations in the spin axis relative to the polar figure axis. The fluctuations in T_v will generate a wobble only if their time scale is of order one month or less. The mean amplitude θ_w in this case should be *ca.* $0.23''\,(2\pi\tau_w/74\,M\,a)^{\frac{1}{2}}$ or 5 to 10 times the mean free nutation amplitude if $\tau_w \approx 10^7$ a. The longitude libration is driven by the component of the fluctuating torque parallel to the spin axis, *ca.* $\sin\epsilon\,T_v$. Although the torque exciting the longitude libration is smaller by a factor of $\sin\epsilon$, the angular momentum in this libration, *ca.* $I_m\theta_L\sigma_L$, is also smaller than that in the nutation, *ca.* $I_m\omega_c\theta_N$, by a factor or σ_L/ω_c. It is found that the mean $\theta_L \approx 0.2''\,(\omega/\sigma_L)\sin\epsilon\,(2\pi\tau_L/3\,M\,a)^{\frac{1}{2}}$. The value of *ca.* 1″ is between the values obtained for θ_N and θ_w, given that $\tau_L \approx 10^5$ a. This crude argument does suggest that fluid turbulence *may* account for the observed librations and their relative amplitudes. This mechanism also predicts that the random fluctuations in the lunar spin axis are of order 0.003″. The present ability to detect the random component of lunar polar motion is marginal at best. Prospects for detecting this signal will improve when lunar laser ranging stations in Australia, Hawaii and Germany obtain ranges of the same quality and number as at McDonald.

Concluding remarks

Turbulent core–mantle friction appears to be a more plausible mechanism than solid tidal friction for the observed offset of the spin axis from the mean Cassini state. There is no evidence from its interior structure or from comparisons with other planetary bodies that supports the requirement that the lunar solid body Q is as low as 28. Fluid turbulence is the primary mechanism coupling the fluid core to the lunar mantle as long as the kinematic viscosity is less than *ca.* 10^2 St. The magnitude of the offset depends principally on the core size. It is found that a 330 km radius core is sufficient to explain the offset and falls within the upper bounds set by Goldstein (1979) and Nakamura *et al.* (1974). Precession-driven turbulence is also a reasonable mechanism for exciting the free figure librations, explaining both the observed relative amplitudes and predicting that the lunar spin axis wanders *ca.* 0.003″ on a time scale of order a month to a few years.

This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract no. NAS 7–100, sponsored by the National Aeronautics and Space Administration. Conversations with Jim Williams, Pat Cassen, David Stevenson, Roger Capallo and Bob King have been helpful. The graphics by Janet Glazier are also gratefully appreciated.

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Discussion

R. Hutchison (*Mineralogy Department, British Museum* (*Natural History*), *Cromwell Road, London SW7 5BD, U.K.*). Contrary to Hartung's (1976) suggestion, the formation of a large impact crater on the Moon probably was not witnessed. In addition to the low probability of the occurrence of such an event in the past millenium, the observations of 1178 are more plausibly explained by the viewing of a bright fireball almost head on, as detailed by Niniger & Huss (1977).

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